

POINT CLOUDS: A REVOLUTIONARY NEW WAY TO SURVEY, DELINEATE AND VISUALISE SUNDIALS

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Imagine a digital camera that can record not just the direction to, and colour of, each of several million points but the exact distance to every point, and thus its position in space. Such images could be processed to calculate how they would look from any other point in space. Well, that is exactly what the latest laser scanners can do, capturing an image that includes the colour and position of up to a million points a second. These images are called point clouds. They are used for the 3D maps on your computer or phone. And, because they are quick and cheap to capture and are accurate to a few millimetres, they are being used for many surveys of buildings, inside and out.

As well as allowing 3D objects such as buildings to be viewed on a computer screen, point clouds can be processed in many ways. Of interest when planning a sundial:

- Scanners can use GPS to record the absolute positions of all points in space.
- Software can calculate the average inclination and declination of any wall or other surface.
- The time at which each point will be shaded by a proposed gnomon can be calculated and the points' colours changed, for instance to draw hour lines or realistic shadows. These hour lines will only be straight if the surface is flat.
- The modified image can be viewed with interactive 3D software on a computer to see how it will look from any viewpoint.
- Positions and dimensions of any feature can be measured.

The point cloud is simply a computer data file that contains an unstructured list of the positions and colours of, typically, millions of points. There is no guarantee that they are in any particular order.

Application of Point Clouds to the Fleet Street Heritage Sundial

Piers Nicholson had, for a long time, identified a large wall at 62 Fleet Street, seen in Fig. 1, as calling out to become a sundial. He had in mind a design that would celebrate the history of Fleet Street, and indeed this very building, as the centre of journalism and printing. The wall faces slightly north of due east and is about 11 metres wide and 16 tall – so large and so high that traditional surveying methods



Fig. 1. Google Maps' 3D view of the site of our sundial. Imagery ©2021 Google.

would have struggled to establish the exact declination of the wall or to verify that it really was flat. So Piers commissioned a point cloud laser survey and asked me to calculate a 10 metre square sundial based on it. Kevin Karney was also on board, charged with planning the layout of the dial which was to feature the mastheads of several newspapers that had once made Fleet Street so famous. By this time Piers had secured funding and the support of the Corporation of the City of London. The story behind all this is fascinating but outside the scope of this article.

Laser scanners incorporate GPS receivers and use advanced techniques to minimise errors, so can record absolute positions and directions with great accuracy. For some purposes absolute position is important, direction not so. For sundials it is the other way round. So I specified that the survey should use true north–south and east–west axes, not magnetic or grid axes. Unfortunately, the surveyors ignored this and it was not until asked to confirm the axes that they admitted they had used the Ordnance Survey's British National Grid. This grid is only accurately aligned to north–south at longitude 2° W. In London the axes are nearly 2° out. The OS publish formulae to calculate the offset at any location, which I was able to use to correct the coordinates.

There are a lot of file formats in which point clouds can be stored. Binary formats are the most compact but hard to

work with so I had asked for an ASCII file in the .PTS format. Although by many standards ours was a modest point cloud, it was still a 2.6 GB file, so the first operation was to discard all points outside our wall. This reduced it to around 9 million points and a mere 500 MB.

These points are roughly 3 mm apart and with an accuracy of a few millimetres, so capture even tiny irregularities in the wall.

When using laser scanners to create point clouds it is common practice to put a distinctive reflector somewhere in the scene to act as a datum from which other features can be measured. We did not have this but identified an architectural feature near the top left corner of the wall that served as an origin for coordinates.

From this point, we suggested that a horizontal line could be drawn across the wall and an approximately vertical one downwards at right angles to it. These could then be used for marking out the dial.

Sundial Calculations on Point Clouds

There exist commercial programs to perform a lot of simple operations on point clouds, but, curiously, none that model sundials. So I wrote all the programs needed to do so, using the language Python. The only other software used was a free point cloud viewer, which was used to create all the following images of the dial.

Using this viewer, it became apparent that the wall is almost perfectly vertical but has a narrow ledge near the top (visible in Fig. 1). The width of the ledge is not constant, implying that the walls above and below are not quite straight. To calculate the parameters of the plane that best fits a large number of points, there exist methods very similar to the well-known least-squares method of fitting a line to a set of points in a plane. Using them, I calculated the inclination and average declinations of the wall areas above and below the ledge, so that Kevin could use traditional sundial design formulae when designing the dial layout. My software could calculate and output hour line coordinates but could not provide the graphics in a form he could use. The design was to be dominated not by hour lines but by huge images of newspaper mastheads, all to be hand drawn. By adjusting the gnomon height and position, selecting 'hour' line times and choosing the best order for the mastheads, Kevin was able to give each title similar prominence and to avoid having the ledge cut any letter in two.

The wall faces just north of due east, so the gnomon needed to be near its top left corner and would be nearly parallel to the wall. The hour lines would all be nearly parallel to one another. Starting from an assumed position for the gnomon I calculated the exact time when each of the 9 million points within the dial would be shaded by it. This is surprisingly easy, using vector mathematics. If we had wanted also to draw declination lines with respect to a nodus, that would have been equally easy.

The Vector Mathematics

This is a digression into the vector mathematics. Skip this section if you wish.

Starting with the assumed position on the wall of one leg of the gnomon and the length of the leg, I first scan the wall area around the leg to determine the normal to the wall at that point, since the leg will be perpendicular to the wall. This gives the 3D coordinates of a point on the gnomon which I will refer to as the nodus, although it is an arbitrary point along the gnomon unless we want to draw its declination lines.

Now, for every point on the dial we have its 3D coordinates and therefore can subtract from them the coordinates of the nodus, giving the vector R of the sun's ray at the moment this point is in the shadow of the nodus. The distance from the dial point to the nodus is given by the magnitude or modulus of R , which I will call D , so $D = \|R\|$.

The axes are due east, due north and vertical. We know our latitude, so the polar gnomon points in direction $P = [0, \sin(\text{lat}), \cos(\text{lat})]$ and the noon equinoctial sun is in direction $N = [0, -\cos(\text{lat}), \sin(\text{lat})]$. The vector dot product of R and P gives the sun's declination at which this point on the dial will be shaded by the nodus, so we could use it to plot declination lines if desired. Simply put:

$$\sin(\text{dec}) = (R \cdot P)/D$$

where $R \cdot P$ is the dot product of R and P .

The hour angle h is 0 at solar noon and increases by 15° per hour. The hour angle at which our point is shaded by the gnomon is found from:

$$\tan(h) = R_E/(R \cdot N)$$

where R_E is the component of R in the east direction.

So, with just one dot product and one arc tangent, we have the exact time at which this point on the dial will be shaded by our polar gnomon. And, without getting involved in hyperbolae, we can draw declination lines, too. We just need to do it for every one of the 9 million points on the dial.

Painting a Sundial on a 3D Image

The first program recoloured any point that was found to be within 1 minute of being on an exact quarter hour (Figs 2, 3). For each hour (including, for instance, 10:30 and 10:45), it also calculated the best-fit straight line to all these points. It then drew these straight lines in purple by adding new points to the point cloud, floating slightly above the wall surface so that they would not get hidden by its irregularities. To label them, I later added Roman numerals with additional points. The 3D viewer lets you see the irregularities of the wall. In the event, all the straight hour lines turned out to be within a couple of minutes of the actual shadow (Fig. 3).

In order to assist the signwriter who was to paint newspaper mastheads between the hour lines, I also

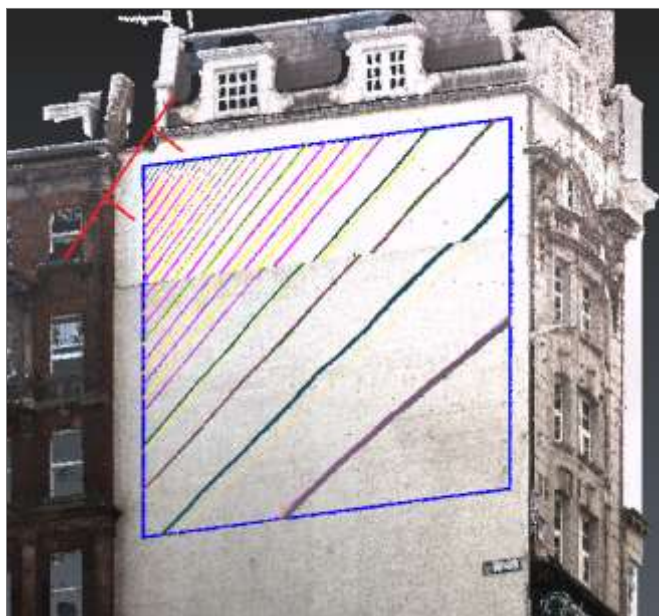


Fig. 2. 3D model with sundial painted on it and gnomon added.



Fig. 3. Oblique view of a true hour 'line' and its best-fit straight line approximation.

calculated and drew the bisectors of the angles between them with yellow lines (Figs 2, 3). However, as we shall see, these had to be abandoned.

Gnomon Design Issues

It had been decided that the gnomon should consist of a stainless steel tube. One design issue with such a large east-facing dial is what diameter to make the gnomon. The human eye is very good at judging the centre of the shadow of a rod or tube. It is very bad at judging the edge of the shadow of a solid, so thin gnomons are best. But if the distance from the tube to the dial is greater than 106 times its diameter the shadow will be all penumbra, which is less distinct than true, fully shaded umbra. So it would be desirable, if possible, to have a gnomon of diameter at least

1/106 of the maximum distance to the dial. But in this case the dial is up to 13 metres from the gnomon, so this calls for a gnomon 12 cm diameter. A steel tube this size was felt to be unacceptably heavy, and the shadow would have been too wide earlier in the morning, so I wrote a program to model the actual shadow of a thinner gnomon.

The way it does this is, for each point in the dial, to find if the gnomon's shadow is at all obscured at that point at any exact quarter hour. If so, we calculate what proportion of the sun is shaded there by the given diameter of gnomon. We then repaint the point white if in full sun, black if fully shaded, the appropriate level of grey in between. The resulting 3D image clearly shows the exact appearance of the dial at every hour. Near the gnomon the shadows are narrow and crisp. Far from it they become wider and less distinct. However, the model gave confidence that even a relatively narrow gnomon should cast a visible shadow on a

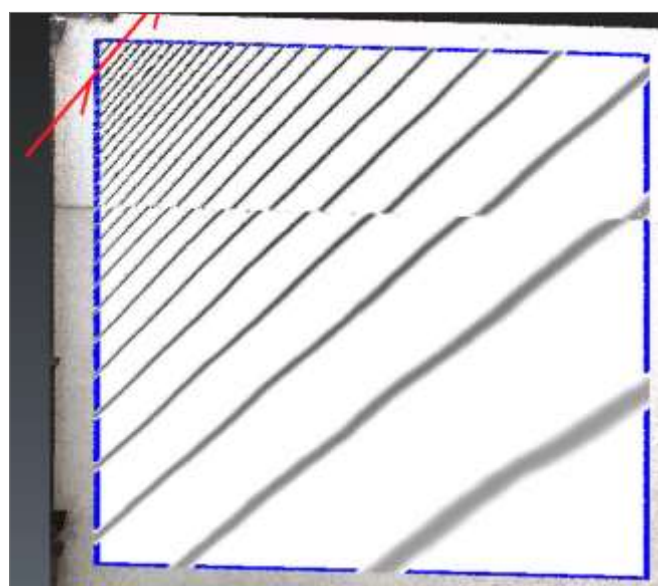


Fig. 4. Accurately calculated shadows, for 3 cm diameter gnomon, every 15 minutes.

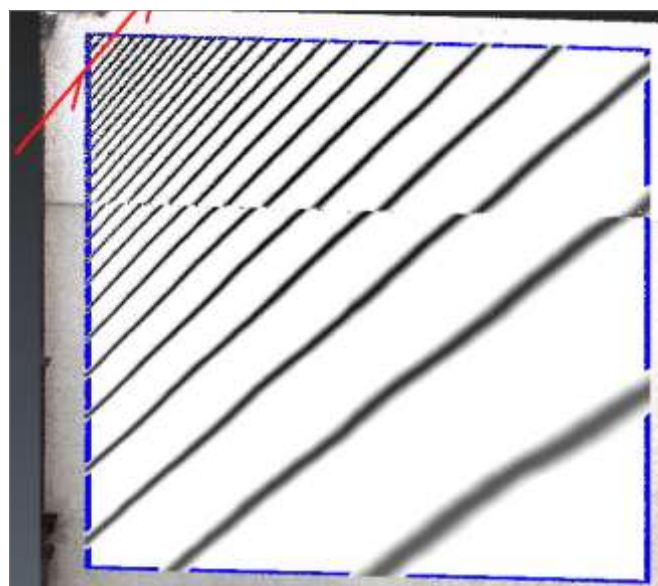


Fig. 5. Accurate shadows for 5 cm diameter gnomon.



Fig. 6. The finished dial seen from Fleet Street at 11:30 am BST on 21 October, when EoT was 15 minutes, so the dial correctly shows $11:30 + 0:15 - 1:00 = 10:45$ sun time.

sunny day (Figs 4, 5). While a fatter gnomon or, perhaps, one with a different cross-section, might be more satisfactory for part of the dial, the chosen design is elegant and works well most of the time.

While the wall is, in many respects, ideal for a monumental sundial, it has, as mentioned, one unfortunate feature: the ledge about 10 metres up and around 4 to 8 cm wide. Viewed by, say, an ant sitting on the gnomon the hour lines look dead straight. But viewed from the ground below they have a distinct ‘N’-shaped kink at the ledge. If we only had

to worry about the hour lines there would be little problem as the kinks are not wide enough to confuse you. But we wanted to fill the gaps between the hour lines with the newspaper mastheads. If these were to replicate the same kinks they would look very odd from Fleet Street, where most passers-by are, since it is to the north. So the signwriter, Steven Whitaker, painted the mastheads to look good from Fleet Street.

Summary

This article has concentrated on just one aspect of this impressive dial – the use of 3D point clouds to survey a site and lay out a sundial. In the author’s opinion point clouds have huge potential to make accurate sundials without the complex mathematics and uncertainties inherent in traditional surveying and layout methods. The proposed dial can be shown on a screen as a 3D model viewable from any position. Accurate dimensions can be measured. Irregularities in the surface can be seen and are automatically accommodated.

There are a lot of companies now offering point cloud surveys as a service. It is very quick and not hugely expensive. I recommend it to you.

The dial, and three plaques accompanying it and explaining it and the history it celebrates, were formally unveiled by the Lady Mayoress of London, Mrs Hilary Russell, on 21 October 2021 (Fig. 6). This is the culmination of a huge amount of work over 14 years by Piers Nicholson, to whom all credit is due. It will be one of the largest timepieces in London – more than twice the area of each Big Ben clock dial.

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